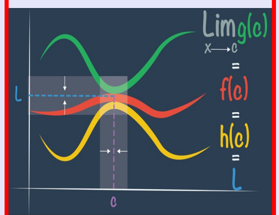
Calculus I

Lecture 18



Feb 19-8:47 AM

Class QZ 13

Given $f'(x) = 5x^4 - 3x^2 + 4$

$f(-1) = 2$

$\boxed{\text{Find } f(x)}$

$\boxed{\text{Box Your final Ans.}}$

$f(x) = x^5 - x^3 + 4x + C$

$f(-1) = (-1)^5 - (-1)^3 + 4(-1) + C = 2$

$\cancel{-1} \; \cancel{+1} \; -4 + C = 2$

$C = 2 + 4$

$\boxed{C = 6}$

$\boxed{f(x) = x^5 - x^3 + 4x + 6}$

Jul 16-7:28 AM

Find $f(x)$

$f''(x) = \sin x + \cos x \qquad f(0)=3 \qquad f'(0)=4$

$f'(x) = -\cos x + \sin x + C$

$f'(0) = -\cancel{\cos 0}^{\to 1} + \cancel{\sin 0}^{\to 0} + C = 4$

$-1 + C = 4 \qquad \boxed{C=5}$

$f'(x) = -\cos x + \sin x + 5$

$f(x) = -\sin x - \cos x + 5x + C$

$f(0) = -\cancel{\sin 0}^{\to 0} - \cancel{\cos 0}^{\to 1} + \cancel{5(0)}^{\to 0} + C = 3$

$-1 + C = 3 \qquad \boxed{C=4}$

$$\boxed{f(x) = -\sin x - \cos x + 5x + 4}$$

Jul 16-8:12 AM

$f''(x) = 20x^3 + 12x^2 + 4$

$f(0)=8 \quad , \quad f(1)=5$

Find $f(x)$

$f''(x) = 5\cdot 4x^3 + 4\cdot 3x^2 + 4$

$f'(x) = 5x^4 + 4x^3 + 4x + C \qquad 4x \to 2\cdot 2x$

$f(x) = x^5 + x^4 + 2x^2 + Cx + D$

$f(0) = \cancel{0^5}^{\to 0} + \cancel{0^4}^{\to 0} + \cancel{2(0)^2}^{\to 0} + \cancel{C(0)}^{\to 0} + D = 8$

$\boxed{D=8}$

$f(x) = x^5 + x^4 + 2x^2 + Cx + 8$

$f(1) = 1^5 + 1^4 + 2(1)^2 + C(1) + 8 = 5$

$1 + 1 + 2 + C + 8 = 5$

$C = 5 - 12$

$\boxed{C=-7}$

$$\boxed{f(x) = x^5 + x^4 + 2x^2 - 7x + 8}$$

Jul 16-8:21 AM

$f'''(x) = \cos x \quad f(0) = 1 \quad f'(0) = 2 \quad f''(0) = 3$

Find $f(x)$.

$f'''(x) = \cos x$

$f''(x) = \sin x + C$

$f''(0) = \cancel{\sin 0}^{\to 0} + C = 3 \quad \boxed{C=3}$

$f''(x) = \sin x + 3$

$f'(x) = -\cos x + 3x + C$

$f'(0) = -\cancel{\cos 0}^{\to 1} + \cancel{3(0)}^{\to 0} + C = 2$

$-1 + C = 2 \quad \boxed{C=3}$

$f'(x) = -\cos x + 3x + 3$

$\frac{d}{dx}\left[\frac{1}{2}x^2\right] = \cancel{\frac{1}{2}} \cdot \cancel{2}x = x$

$f(x) = -\sin x + 3 \cdot \frac{1}{2}x^2 + 3x + C$

$f(x) = -\sin x + \frac{3}{2}x^2 + 3x + C$

$f(0) = -\cancel{\sin 0}^{\to 0} + \cancel{\frac{3}{2}(0)^2}^{\to 0} + \cancel{3(0)}^{\to 0} + C = 1 \quad \boxed{C=1}$

$$\boxed{f(x) = -\sin x + \frac{3}{2}x^2 + 3x + 1}$$

Jul 16-8:31 AM

$f(x) = \dfrac{x^2}{x^2+3}$

1) Domain $(-\infty, \infty)$

$x^2 + 3 \neq 0$

2) V.A. None

3) All intercepts $(0,0)$

4) Even, odd, or neither?

$f(-x) = \dfrac{(-x)^2}{(-x)^2+3} = \dfrac{x^2}{x^2+3} = f(x)$

Even $\to$ Symmetric w/t Y-axis.

5) H.A. $\lim\limits_{x \to \infty} f(x) = \lim\limits_{x \to \infty} \dfrac{x^2}{x^2+3} = \lim\limits_{x \to \infty} \dfrac{1}{1+\frac{3}{x^2}} = 1$

$\lim\limits_{x \to -\infty} f(x) = 1 \Rightarrow$ H.A. $y = 1$

Jul 16-8:39 AM

6) Find $f'(x)$, find all C.P.

$$f(x) = \frac{x^2}{x^2+3} = \frac{x^2+3-3}{x^2+3} = \frac{x^2+3}{x^2+3} - \frac{3}{x^2+3}$$

$$= 1 - 3(x^2+3)^{-1}$$

$$f'(x) = 0 - 3\cdot(-1)(x^2+3)^{-2}\cdot 2x = \frac{6x}{(x^2+3)^2}$$

$f'(x) = 0 \rightarrow 6x = 0 \rightarrow x = 0 \rightarrow$ C.P. $(0,0)$

$f'(x)$ is never undefined $(x^2+3 \neq 0)$

Jul 16-8:46 AM

7) Find $f''(x)$, find all possible inflection pts

$$f'(x) = \frac{6x}{(x^2+3)^2}$$

$$f''(x) = 6\left[\frac{1(x^2+3)^2 - x\cdot 2(x^2+3)^1\cdot 2x}{(x^2+3)^4}\right]$$

$$= 6\left[\frac{\cancel{(x^2+3)}[x^2+3-4x^2]}{(x^2+3)^{\cancel{4}\,3}}\right] = \frac{6(3-3x^2)}{(x^2+3)^3}$$

$$= \frac{6\cdot 3(1-x^2)}{(x^2+3)^3} = \frac{18(1-x)(1+x)}{(x^2+3)^3}$$

P.I.P.

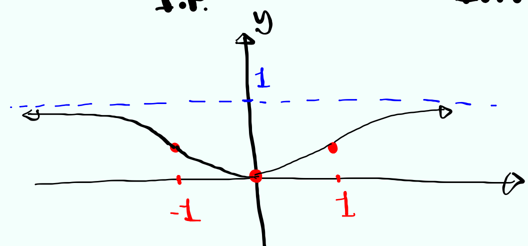
$f''(x) = 0 \rightarrow 18(1-x)(1+x) = 0 \rightarrow x = 1, x = -1$

$f''(x)$ is never undefined $(x^2+3 \neq 0)$

Jul 16-8:51 AM

$f'(x) = \dfrac{6x}{(x^2+3)^2}$ $\qquad$ $f''(x) = \dfrac{18(1-x)(1+x)}{(x^2+3)^3}$

x	$-\infty$		-1		0		1		∞
$f'(x)$		$-$		$-$		$+$		$+$	
$f''(x)$		$-$		$+$		$+$		$-$	
$f(x)$			I.P.				I.P.		



$f(-1) = \frac{1}{4}$

Range $[0, 1)$

$f(1) = \frac{1^2}{1^2+3} = \frac{1}{4}$

Jul 16-8:58 AM

$f(x) = x^5 - 5x$

1) Domain $(-\infty, \infty)$

2) Y-Int $(0, 0)$

3) x-Ints $(0,0)$ $(\pm\sqrt[4]{5}, 0)$

$f(x) = 0$

$x^5 - 5x = 0$ $\qquad$ $x(x^4 - 5) = 0$

$x = 0 \quad , \quad x^4 = 5$

$\boxed{x = \pm\sqrt[4]{5}}$

4) Even, odd, or neither

$\boxed{f(-x)} = (-x)^5 - 5(-x)$

$= -x^5 + 5x$

$= -(x^5 - 5x)$

$= \boxed{-f(x)}$ odd

sym. w/t the origin

Jul 16-9:07 AM

5) $f'(x)$, find C.N., find C.P. $f(x)=x^5-5x$

$f'(x)=5x^4-5$

$=5(x^4-1)$

$=5(x^2+1)(x^2-1)$

$=5(x^2+1)(x+1)(x-1)$

$5x^4-5=0$

$x^4-1=0$

$x=\pm\sqrt[4]{1}$

$x=\pm 1$

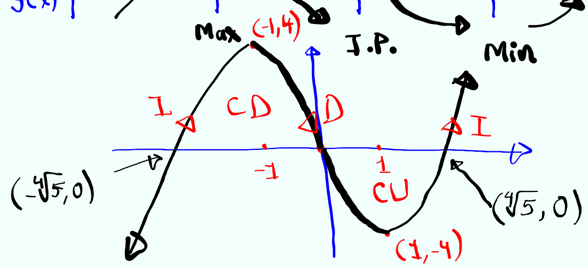
$(1,-4)$

$(-1,4)$

6) $f''(x)$ Find all P.I.P.

$f''(x)=20x^3$ $\quad 20x^3=0$ $\quad x=0$ $\quad (0,0)$

x	$-\infty$		-1		0		1		∞
$f'(x)$		$+$		$-$		$-$		$+$	
$f''(x)$		$-$		$-$		$+$		$+$	
$f(x)$									



Jul 16-9:12 AM

Estimate $\sqrt{17}$ using Quadratic Approximation

Using Calc.

$\sqrt{17} \approx \boxed{4.1231}$

$f(x)=\sqrt{x}$

$a=16$

$f(x)=x^{1/2}$

$f'(x)=\frac{1}{2}x^{-1/2}$

$f''(x)=\frac{-1}{4}x^{-3/2}$

$$f(x)\approx f(a)+f'(a)(x-a)+\frac{f''(a)}{2}(x-a)^2$$

$$\sqrt{x}\approx 4+\frac{1}{8}(x-16)+\frac{\frac{-1}{256}}{2}(x-16)^2$$

$$\approx 4+\frac{1}{8}(x-16)-\frac{1}{512}(x-16)^2$$

$f(a)=\sqrt{16}=\boxed{4}$

$f'(a)=\frac{1}{2}(16)^{-1/2}=\frac{1}{2}\cdot\frac{1}{\sqrt{16}}=\boxed{\frac{1}{8}}$

$f''(a)=\frac{-1}{4}(16)^{-3/2}=\frac{-1}{4(\sqrt{16})^3}=\frac{-1}{4\cdot 64}=\boxed{\frac{-1}{256}}$

$\sqrt{17}\approx 4+\frac{1}{8}-\frac{1}{512}$

$\approx\boxed{4.1230}$

Jul 16-9:57 AM

$f(x)=\frac{1}{x}$, $[1,3]$

check conditions of MVT, then find all c in $(1,3)$ that is in the conclusion of MVT.

1) $f(x)$ is cont. on $[1,3]$ ✓ $f'(x)=\frac{-1}{x^2}$

2) $f(x)$ is diff. on $(1,3)$ ✓

By MVT, there is at least one number c in $(1,3)$ such that $f'(c)=\frac{f(b)-f(a)}{b-a}$

$\frac{-1}{c^2}=\frac{f(3)-f(1)}{3-1}$ $\frac{-1}{c^2}=\frac{\frac{1}{3}-\frac{1}{1}}{2}$

$\frac{\cancel{-}1}{c^2}=\frac{\frac{\cancel{-}2}{3}}{2}$ $\frac{1}{c^2}=\frac{2}{6}$ $\frac{1}{c^2}=\frac{1}{3}$

$c^2=3$ $c=\pm\sqrt{3}$

$\boxed{c=\sqrt{3}}$ is in $(1,3)$

Jul 16-10:07 AM

$f(x)=\sqrt[3]{x}$, $[0,1]$

check the conditions of MVT, then find all numbers c in $(0,1)$ that is in the conclusion of MVT.

$f(x)=x^{1/3}$

$f'(x)=\frac{1}{3}x^{-2/3}$

1) $f(x)$ is cont. on $[0,1]$ ✓

2) $f(x)$ is diff. on $(0,1)$ ✓ $f'(x)=\frac{1}{3\sqrt[3]{x^2}}$

By MVT $f'(c)=\frac{f(b)-f(a)}{b-a}$

$\frac{1}{3\sqrt[3]{c^2}}=\frac{f(1)-f(0)}{1-0}$ $\frac{1}{3\sqrt[3]{c^2}}=\frac{1-0}{1-0}$

$\frac{1}{3\sqrt[3]{c^2}}=1$

$3\sqrt[3]{c^2}=1$

$\sqrt[3]{c^2}=\frac{1}{3}$

$(\sqrt[3]{c^2})^3=\left(\frac{1}{3}\right)^3$

$c^2=\frac{1}{27}$

$c=\pm\sqrt{\frac{1}{27}}$

$c=\frac{1}{\sqrt{27}}=\frac{1}{3\sqrt{3}}=\boxed{\frac{\sqrt{3}}{9}}$

$\approx\boxed{.192}$ → in in $(0,1)$

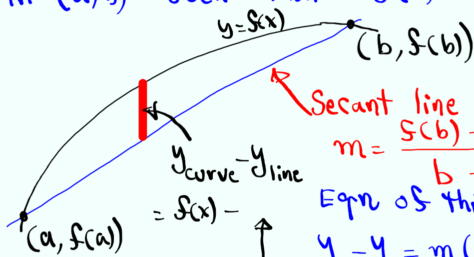
Jul 16-10:14 AM

MVT

1) $f(x)$ is cont. on $[a,b]$,

2) $f(x)$ is diff. on (a,b)

then there is at least a number c in (a,b) such that $f'(c) = \dfrac{f(b)-f(a)}{b-a}$.



$= f(x) -$

Secant line

$m = \dfrac{f(b)-f(a)}{b-a}$

Eqn of this line

$y - y_1 = m(x - x_1)$

$y - f(a) = \dfrac{f(b)-f(a)}{b-a}(x-a)$

$$\boxed{y = \frac{f(b)-f(a)}{b-a}(x-a) + f(a)}$$

eqn of the Secant line

$H(x) = f(x) - \dfrac{f(b)-f(a)}{b-a}(x-a) - f(a)$

Jul 16-10:24 AM

$$H(x) = f(x) - \frac{f(b)-f(a)}{b-a}(x-a) - f(a)$$

1) $H(x)$ is cont. on $[a,b]$,

2) $H(x)$ is diff. on (a,b),

3) $H(a) = \cancel{f(a)} - \dfrac{f(b)-f(a)}{b-a}\underbrace{(a-a)}_{0} - \cancel{f(a)} = 0$

$H(b) = f(b) - \dfrac{f(b)-f(a)}{\cancel{b-a}}\cancel{(b-a)} - f(a)$

$= \cancel{f(b)} - \cancel{f(b)} + \cancel{f(a)} - \cancel{f(a)} = 0$

$H(a) = H(b)$

$H(x)$ is cont. on $[a,b]$, $H(x)$ is diff. on (a,b), $H(a) = H(b)$

by Rolle's thrm there is at least a number c in (a,b) such that $H'(c) = 0$

Jul 16-10:33 AM

$$H(x) = f(x) - \frac{f(b)-f(a)}{b-a}(x-a) - f(a)$$

$$H'(x) = f'(x) - \frac{f(b)-f(a)}{b-a} \cdot 1 - 0$$

$$H'(x) = f'(x) - \frac{f(b)-f(a)}{b-a}$$

By Rolle's Thrm

$H'(c) = 0$ $\quad f'(c) - \frac{f(b)-f(a)}{b-a} = 0$

Conclusion of MVT $\quad \boxed{f'(c) = \frac{f(b)-f(a)}{b-a}}$

Jul 16-10:41 AM

Evaluate $\lim\limits_{x \to \infty} \frac{x}{\sqrt{4x^2+1}}$ $\quad$ as $x \to \infty$ $\quad x = \sqrt{x^2}$

Divide by x

$$\lim_{x \to \infty} \frac{\frac{x}{x}}{\sqrt{\frac{4x^2+1}{x^2}}} = \lim_{x \to \infty} \frac{1}{\sqrt{4 + \frac{1}{x^2}}} \xrightarrow{} 0 = \frac{1}{\sqrt{4}} = \boxed{\frac{1}{2}}$$

Now $\lim\limits_{x \to -\infty} \frac{x}{\sqrt{4x^2+1}}$ $\quad$ as $x \to -\infty$ $\quad x = -\sqrt{x^2}$

Divide by x

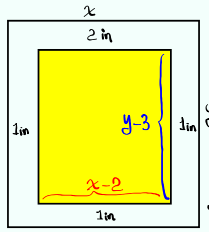
$$= \lim_{x \to -\infty} \frac{\frac{x}{x}}{-\sqrt{\frac{4x^2+1}{x^2}}} = \lim_{x \to -\infty} \frac{1}{-\sqrt{4 + \frac{1}{x^2}}} \xrightarrow{} 0 = \boxed{\frac{-1}{2}}$$

HA. $y = \frac{1}{2}$, $y = \frac{-1}{2}$

Jul 16-10:44 AM

A poster is to have an area of 180 in^2
with 1-in margin at the bottom,
2-in margin at the top, and
1-in margin on the sides.

Find dimensions of the largest printed area



$xy = 180 \text{ in}^2$.

$y = \frac{180}{x}$

Printed Area

$(x-2)(y-3)$

Maximum

$f(x) = (x-2)\left(\frac{180}{x} - 3\right)$

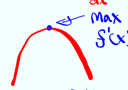
$f(x) = 180 - 3x - \frac{360}{x} + 6$

$\frac{d}{dx}\left[\frac{1}{x}\right] = \frac{-1}{x^2}$

$f(x) = 186 - 3x - \frac{360}{x}$

$f'(x) = 0 - 3 + \frac{360}{x^2}$

$f''(x) = \frac{-720}{x^3} < 0$ CD



$f'(x) = 0$

$-3 + \frac{360}{x^2} = 0$

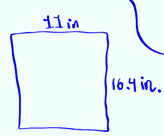
$\frac{360}{x^2} = 3$

$3x^2 = 360$

$x^2 = 120$

$x = \sqrt{120}$

$x \approx 11$ in.



Jul 16-10:52 AM